CTNT 2024, JUNE 14 - 16
SCHEDULE, TITLES, AND ABSTRACTS

TITLES AND ABSTRACTS (Alphabetical by last name)

Deewang Bhamidipati (UC Santa Cruz)

Title: Strata intersections in unitary Shimura varieties of low signature
Abstract: Unitary Shimura varieties are moduli spaces of abelian varieties with a certain extra structure, which includes a signature condition. A useful way to understand these spaces is by stratifying them, of which two are of interest: the Ekedah-Oort (EO) stratification, defined with respect to the p-torsion group scheme structure up to isomorphism; and the Newton stratification, define with respect to the p-divisible group structure up to isogeny. We take a specific stratum in the Newton stratification – the supersingular stratum – and we study its intersection with the EO stratification in the case of signature $(q-2, 2)$. This is joint work with Emerald Anne, Maria Fox, Heidi Goodson, Steven Groen and Sandra Nair.

Bryden Cais (University of Arizona)

Title: Iwasawa-theoretic analogues of the Riemann–Hurwitz and Deuring–Shafarevich formulae
Abstract: A fundamental numerical invariant of a smooth projective curve X over a field k is its genus. In any branched Galois cover of such curves $Y \to X$ over k with group G, the Riemann–Hurwitz formula expresses the genus of Y in terms of the genus of X, the order of G, and the ramification of the cover. When k has positive characteristic p, the Frobenius endomorphism of the cohomology group $H^1(O_X)$ provides a litany of refined numerical invariants of X. In this setting, when G is a p-group, the Deuring–Shafarevich formula is a beautiful analogue of the Riemann–Hurwitz formula for p-ranks, i.e., the dimension of the maximal F-stable subspace of $H^1(O_X)$ on which F is invertible. One would like an analogue of these celebrated formulae for the "rest" of $H^1(O_X)$, on which F acts nilpotently. Simple examples show that there can be no such exact formula in general. Nevertheless, in recent joint work with Booher, Kramer–Miller and Upton, we establish a new, Iwasawa-theoretic analogue of the Riemann–Hurwitz and Deuring–Shafarevich formulae in certain ramified Z_p-extensions of the rational function field over a finite field of characteristic p. This talk will give a survey of this work and some related ideas and questions.

Harris Daniels (Amherst College)

Title: Near Coincidences and Nilpotent Division Fields of Elliptic Curves
Abstract: A natural question one can ask about the division fields of a fixed elliptic curve is whether or not there exists distinct integers m and n such that the m-division field is equal to the n-division field. In 2023, Daniels and Lozano-Robledo published partial answers to this question and one of the main techniques they used was showing that the nth roots of unity couldn't be in the m-division

field. Thus making it impossible for the n- and m-division fields to coincide. Inspired by this, we ask the following more general question about "near coincidences" of division fields: When is there an elliptic curve $E/\mathbb{Q}$ and distinct integers m and n such that $\mathbb{Q}(E[n]) = \mathbb{Q}(E[m], \zeta_n)$? In the first half of this talk we will present an answer to this question in the case when m and n are powers of the same prime.

We then turn our attention to a seemingly unrelated question. We ask for a complete classification of elliptic curves $E/\mathbb{Q}$ and positive integers n such that $\mathrm{Gal}(\mathbb{Q}(E[n])/\mathbb{Q})$ is a nilpotent group. This is a natural question to ask in the face of the classification of abelian division fields provided by Gonzalez Jimenez and Lozano-Robledo in 2016. We then use our previous result to find a conditionally complete classification of nilpotent division fields of elliptic curves. Our classification relies either on an increasingly standard conjecture about the rational points on modular curves associated to the normalizers of non-split Cartan groups, or a complete classification of the Mersenne primes. This is joint work with Jeremy Rouse.

Seong Eun Jung (UMass Amherst)

Title: Modular symbols over function fields of elliptic curves
Abstract: Created by Manin, modular symbols are classes of paths on $\mathbb{P}^1(\mathbb{Q})$: elements of the relative homology group $H_1(X, \text{cusps})$ where X is the modular curve, the quotient of a congruence subgroup of $\mathrm{SL}(2,\mathbb{Z})$ on the complex upper half plane. He was able to find the explicit generators for this group as well as the complete set of relations. Later, Teitelbaum constructed modular symbols over the rational function field $\mathbb{F}_q(T)$. He was able to define modular symbols and the complete set of relations for this case. Building off of Teitelbaum's work, we look at what happens over function fields of elliptic curves. We first construct the analogs of the complex upper half plane, $\mathrm{SL}(2,\mathbb{Z})$, and the modular curve X. Then we define modular symbols as well as the relations among them over this setting.

Tyler Genao (The Ohio State University)

Title: New isogenies of elliptic curves over number fields
Abstract: In analogy to understanding torsion groups of elliptic curves, it is of great interest to understand the rationality of isogenies over various number fields. For example, work of Mazur has shown that any $\mathbb{Q}$-rational isogeny of prime degree ℓ on an elliptic curve $E_{/\mathbb{Q}}$ satisfies $\ell \in \{2,3,5,7,11,13,17,19,37,43,67,163\}$. With our current understanding of elliptic curve Galois representations over $\mathbb{Q}$, one can also show that any isogeny on an elliptic curve over $\mathbb{Q}$ is either $\mathbb{Q}$-rational, or has a field of definition whose degree is a multiple of one of the primes above. In particular, no "new" isogenies of elliptic curves over $\mathbb{Q}$ can appear over a number field whose degree is coprime to these numbers.

In this talk, I will introduce a generalization of this result on new isogenies: if a number field F has no "rational complex multiplication," then under GRH there exists a constant $B := B(F) \in \mathbb{Z}^+$ such that for any finite extension L/F whose degree $[L : F]$ is coprime to B, one has for all elliptic curves over F with j-invariant $\neq 0, 1728$ that any L-rational cyclic isogeny on E must already be F-rational. Towards explaining this, I will also introduce a uniformity result on the "relative largeness" of mod-ℓ Galois representations of elliptic curves over F when ℓ is uniformly large, à la work of Serre. I will

also describe unconditional results for the mod-ℓ Galois representations of non-CM elliptic curves with an F-rational ℓ-isogeny when ℓ is uniformly large.

Summer Haag (CU Boulder)

Title: An Overview of the Local-to-Global Conjecture for Apollonian Circle Packings
Abstract: Recently, the "local-to-global" conjecture for Apollonian circle packings was disproved for certain packings. In this talk, I will outline the basics of Apollonian circle packings, such as Descartes quadruples and curvature obstructions. I will then discuss the evidence for the conjecture and how the conjecture fails, primarily the computational evidence and how quadratic reciprocity is used in circle packings.

Sachi Hashimoto (Brown University)

Title: Local heights and rational points
Abstract: The method of quadratic Chabauty is a powerful tool for determining the set of rational points on a curve. A central part of this method are p-adic height functions. In this talk, I will present an algorithm to compute local p-adic heights at odd primes ℓ not equal to p on hyperelliptic curves. I will highlight examples and applications to Diophantine problems. This is joint work with Alexander Betts, Juanita Duque Rosero, and Pim Spelier.

Colette LaPointe (CUNY Graduate Center)

Title: Bifurcations and discriminants for rational maps
Abstract: Bifurcation points of a parametrized family of maps are values of the parameter at which a dramatic shift in the dynamical behavior of the associated maps occurs. For example, bifurcation has frequently been studied for the family $f_c = z^m + c$, with $m \geq 2$, as the parameter c is varied over the complex numbers. Bifurcation points can also be thought of as the maps in the family that have colliding periodic orbits. In 1994, Morton and Vivaldi gave an algebraic description of the bifurcation points of any family of monic polynomials as the zeros of the discriminants of the dynatomic polynomials, i.e. polynomials whose roots form the periodic orbits of a dynamical system. In this talk, I will discuss my generalization of their results to the case of rational maps of degree $d \geq 2$ on $\mathbb{P}^1(k)$ for a field k.

Kyu-Hwan Lee (University of Connecticut)

Title: AI-assisted mathematical discovery: murmurations of elliptic curves
Abstract: Recently, there have been successful attempts to apply machine learning to various objects in pure mathematics. After overviewing these activities, we will focus on the case of elliptic curves, where a new phenomenon, called *murmuration*, has been discovered through machine learning techniques in my collaboration with He, Oliver and Pozdnyakov. Various efforts to understand this new phenomenon resulted in several recent papers by a number of people.

Brody Lynch (UMass Amherst)

Title: Distribution of Steinitz Classes for prime degree Kummer extensions.
Abstract: Let K be a number field. It is well-known that $\mathcal{O}_K$, the ring of integers of K, is a free $\mathbb{Z}$-module. However, if L/K is a finite extension, in general $\mathcal{O}_L$ need not be a free $\mathcal{O}_K$-module. The existence of a relative integral basis is determined by the Steinitz class of the extension. Using the deep theory of Shintani zeta functions, Kable and Wright prove that Steinitz classes of relative quadratic extensions of K are uniformly distributed across each element of the class group. In this talk, we reprove this theorem using only classical algebraic number theory. Then we generalize our methods to investigate the distribution of Steinitz classes for prime degree Kummer extensions, which is not yet known.

Jacob Mayle (Wake Forest University)

Title: Average congruence class biases in the cyclicity and Koblitz conjectures
Abstract: Given an elliptic curve over the rationals, it is natural to ask about the distribution of primes p for which the reduction of E modulo p has certain properties. Two well-known problems of this nature are the cyclicity and Koblitz problems, which ask about the primes of cyclic and prime order reduction, respectively. In this talk, we'll discuss variants of these problems that consider primes in arithmetic progression. In particular, we'll highlight a somewhat counterintuitive phenomenon: the primes of cyclic reduction and prime order reduction for elliptic curves are oppositely biased on average over congruence classes. This is a joint work with Sung Min Lee and Tian Wang.

James Austin Myer (CUNY Graduate Center)

Title: (Toward) An Algorithm to (Explicitly) Produce a Regular Model of a Hyperelliptic Curve in (Bad) Characteristic (0, 2): A Criterion to Verify Regularity of the Normalization of a Candidate Model
Abstract: Given a hyperelliptic curve (defined over a "pleasant" field of characteristic 0 whose ring of integers is of (bad) mixed characteristic (0, 2)), we seek a regular model, i.e. a(n arithmetic) surface fibered over the (spectrum of the) ring of integers of the field whose generic fiber is the given curve, and with a special fiber: its avatar in characteristic 2. A strategy is afforded within a paper of Dino Lorenzini & Qing Liu: there exists a (regular) model of the projective line whose normalization in the function field of the given hyperelliptic curve is its sought after regular model. So, we seek such a regular model of the projective line... A candidate such model is gifted to us (explicitly) by work of Andrew Obus & Padmavathi Srinivasan. We establish a stepping stone across the river toward an algorithm to (explicitly) produce a regular model of any hyperelliptic curve in (bad) mixed characteristic (0, 2): a criterion to verify the regularity of the normalization of a candidate model of a hyperelliptic curve (equivalently, the normalization of the candidate model of the projective line of Obus & Srinivasan in the function field of the hyperelliptic curve).

Jennifer Park (The Ohio State University)

Title: Intermediate Hilbert's Tenth Problems
Abstract: It is well-known that Hilbert's tenth problem, which asks for an algorithm to detect whether a polynomial equation has integer solutions or not, is undecidable, by the works of Matiyasevich and Davis–Putnam–Robinson. On the other hand, a variant of Hilbert's tenth problem, where one searches for rational solutions to a polynomial instead of integer solutions, is still open (that is, we don't know whether this problem is undecidable). In this talk, I will describe some attempts towards Hilbert's tenth problem over $\mathbb{Q}$; in particular, by considering the intermediate problems that could potentially link this problem to the original Hilbert's tenth problem.

Freddy Saia (University of Illinois Chicago)

Title: Bielliptic Shimura curves $X_0^D(N)$
Abstract: Since Mazur's work on rational isogenies and rational torsion of elliptic curves over $\mathbb{Q}$, there has been concerted effort towards classifying (infinitude of) low degree points on modular curves such as $X_0(N)$ and $X_1(N)$. Considerably less is known for Shimura curves, which parameterize abelian surfaces with quaternionic multiplication and analogous torsion structures. By a result of Shimura, these curves have no real points, hence no odd degree points, so we train our focus first at degree 2. We will discuss the determination of the Shimura curves $X_0^D(N)$ with infinitely many quadratic points, resulting from a study of the bielliptic curves in this family. This is based on joint work with Oana Padurariu.

Padma Srinivasan (Boston University)

Title: A canonical algebraic cycle associated to a curve in its Jacobian
Abstract: How many subvarieties of a given dimension does an algebraic variety have? What are some natural ways to algebraically "deform" one subvariety to another? The best understood case is that of 0-dimensional subvarieties of a 1-dimensional variety – points on curves! – and goes back to classical work of Abel and Jacobi. In higher dimensions, there are a few natural generalizations of the notion of deformation equivalence, but it's unclear if these different notions are the same! In the 1980s, Ceresa exhibited one of the first naturally occurring examples of an algebraic cycle, the Ceresa cycle, which shows the notions of homological and algebraically equivalence can be different on higher dimensional varieties. In this talk, we will survey many recent results about the Ceresa cycle and introduce arithmetic and geometric tools that can be used to certify its nontriviality.

Isabel Vogt (Brown University)

Title: Brauer–Manin obstructions requiring arbitrarily many Brauer classes
Abstract: A fundamental problem in the arithmetic of varieties over global fields is to determine whether they have a rational point. As a first effective step, one can check that a variety has local points for each place. However, this adelic information is not enough, as many classes of varieties are known to fail the local-global principle. The Brauer–Manin obstruction to the local-global principle for rational points comes out of class field theory and is captured by elements of the Brauer group.

On a projective variety, any Brauer–Manin obstruction is captured by a finite subgroup of the Brauer group. I will start by introducing this framework for understanding some failures of the local-global principle and then move on to explain joint work that shows that this subgroup can require arbitrarily many generators. This is joint with J. Berg, C. Pagano, B. Poonen, M. Stoll, N. Triantafillou and B. Viray.

Jeffrey Yelton (Wesleyan University)

Title: Clusters and non-archimedean uniformization of superelliptic curves
Abstract: Let K be a field with a discrete valuation; let S be an even-cardinality subset; and choose a prime p. Under suitable conditions, such a set S may be used to determine a group of fractional linear transformations whose action on $\mathbb{P}^1_K$ induces a quotient that can be algebraized as a superelliptic curve $C : y^p = f(x)$ and such that the quotient map induces a bijection between S and the set $\mathcal{B}$ of roots of f. I will describe how one may one test whether any given even-cardinality subset S has the right properties to induce a curve in this way, and in the case that S does satisfy these properties, how the cluster data of S (the non-archimedean distances between the pairs of its points) compare to those of its image $\mathcal{B}$.

Benjamin York (University of Connecticut)

Title: Models of CM elliptic curves with prescribed ℓ-adic Galois image
Abstract: In 2018, Lozano-Robledo provided a classification for ℓ-adic Galois representations attached to elliptic curves with complex multiplication (CM). In this talk, we will discuss a classification of Weierstrass models for CM elliptic curves with specified ℓ-adic Galois representation, and discuss our methods for proving this classification. This is joint work with Enrique González-Jiménez and Álvaro Lozano-Robledo.

Joshua Zelinsky (Hopkins School)

Title: On strongly pseudoperfect numbers and the arithmetic-geometric mean inequality
Abstract: We discuss strongly pseudoperfect numbers, numbers n which are pseudoperfect and where they have pseudoperfect sum set S, such that $d \in S$ if and only if $\frac{n}{d} \in S$. We discuss how many properties of perfect numbers which do not apply to all pseudoperfect numbers do apply to strongly pseudoperfect numbers. A major role is played by the arithmetic-mean-geometric mean inequality.

Robin Zhang (Massachusetts Institute of Technology)

Title: The Stark conjecture in a modular* setting
Abstract: The special value $\zeta(0)$ of Riemann zeta function was calculated to be $-\frac{1}{2}$ by Euler in 1740. The Dedekind zeta function is a generalization of the Riemann zeta function to any number field K; the leading coefficient of its Taylor series at $s = 0$ is precisely described in terms of units and the class number of K in the class number formula. The Artin L-function is a further generalization to any Galois representation ρ of $\mathrm{Gal}(K/\mathbb{Q})$; its leading coefficient at $s = 0$ is predicted to be

given by units and algebraic invariants of ρ in the Stark conjecture. Very few cases of the Stark conjecture have been proved in the last 50 years, but a recent flurry of activity has led to many of the refinements & analogues (modulo* p, p-adic, and function field) formulated by Brumer, Tate, Gross, Rubin, Popescu, Kurihara, and others being resolved in the last decade. I'll report on new developments in the Harris–Venkatesh conjecture about the action of derived Hecke operators on modular* forms of weight 1 and the questions that they raise for the Stark conjecture.

David Zureick-Brown (Amherst College)

Title: "Sporadic" torsion on elliptic curves
Abstract: I'll survey various recent results about torsion points on elliptic curves, especially "sporadic" (or "unexpected") torsion points over fields other than the rationals.